E 3851 (Pages : 3) Reg. No...............................

Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2016

Third Semester

Core Course 3—CALCULUS

(Common for Model I, Model II Mathematics and B.Sc. Computer Applications)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A

*Answer **all** questions from this part.*
Each question carries 1 mark.

1. Write the n^{th} derivative of $\sin(ax+b)$.
2. State Taylor's theorem.
3. Define the terms evolute and involute.
4. Test whether $f(x,y)=\sin\left(\dfrac{x^3+y^3}{x-y}\right)$ is a homogeneous function.
5. Define Saddle points.
6. If $z=x^y$, find $\dfrac{\partial z}{\partial y}$.
7. Write the shell formula for obtaining the volume of solid by revolving the region about y-axis.
8. State Pappus's theorem for surface area.
9. Write the co-ordinate conversion formula from spherical to cylindrical co-ordinates.
10. Define triple integral of a function $f(x,y,z)$ over a bounded region in space.

(10 × 1 = 10)

Part B

*Answer any **eight** questions.*
Each question carries 2 marks.

11. Show that the curve $y=\dfrac{6x}{3+x^2}$ has three points of inflexion.

Turn over

12. Find the n^{th} derivative of $\log\left(9x^2-4\right)$.

13. Assuming the possibility of expansion prove that $\log\cosh x = \frac{x^2}{2} - \frac{x^4}{12} + \frac{x^6}{45} \ldots$

14. If $u = \sin(xy)$, show that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$.

15. Verify Euler's theorem for $u = x^3 - y^3 + 3x^2y$.

16. Find the asymptotes parallel to the co-ordinate axes of the curve $x^4 + x^2y^2 - a^2\left(x^2+y^2\right) = 0$.

17. Find the area of the region enclosed by $y = x^2 - 2x$ and $y = x$.

18. Find the volume of the solid generated by revolving the region bounded by $y = x^2, y = 0, x = 2$ about the x-axis.

19. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$.

20. Integrate $f(x,y) = x/y$ over the region bounded by the lines $y = x, y = 2x, x = 1, x = 2$ in the first quadrant.

21. Write an equivalent double integral with the order of integration reversed for $\int_0^1 \int_2^{4-2x} dy\, dx$.

22. Evaluate $\int_0^1 \int_0^{1-z} \int_0^2 dx\, dy\, dz$.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*

Each question carries 4 marks.

23. If $\cos^{-1}(y/b) = n\log(x/n)$, prove that $x^2y_{n+2} + (2n+1)x\, y_{n+1} + 2n^2y_n = 0$.

24. Find the radius of curvature for $y^2 = (x+4)x^2$ of the points where the tangent is parallel to the x-axis.

25. Find the envelope of the curve $y^2 = 4a(x-a)$.

26. If $V = \left(x^2 + y^2 + z^2\right)^{-1/2}$, show that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$.

27. A rectangular box, open at the top, is to have a volume of 32 cubic feet. What must be the dimensions so that the total surface is a minimum ?

28. A pyramid 3 m high has a square base that is 3 m on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.

29. Find the area of the region enclosed by $y = 2\sin x$ and $y = \sin 2x, 0 \le x \le \pi$.

30. Find the volume of the prism whose base is the triangle in the xy plane bounded by the x-axis and the lines $y = x$ and $x = 1$. Where top lies in the plane $z = 3 - x - y$.

31. Find the area enclosed by one leaf of the rose $r = 12\cos 3\theta$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Find the co-ordinates of the centre of curvature of the point $x = at^2, y = 2at$ on the parabola $y^2 = 4ax$ and hence find its evolute.

(b) Derive the expansion of $\log(1+\sin x)$ in the form $x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} \cdots$

33. Find the maximum of $(x_1, x_2, \ldots, x_n)^2$ with the condition that $x_1^2 + x_2^2 + \ldots x_4^2 = 1$ using the method of Lagrange's multipliers.

34. (a) Find the area of the surface generated by revolving the curve $y = x^3, 0 \le x \le \frac{1}{2}$ about the x-axis.

(b) The region in the first quadrant bounded by the parabola $y = x^2$, the y-axis and the line $y = 1$ is revolved about the line $x = 2$ to generate a solid. Find the volume of the solid.

35. Evaluate $\int_0^4 \int_{x=y/2}^{x=\frac{y}{2}+1} \frac{2x-y}{2}\, dx\, dy$ by applying the transformation $u = \frac{2x-y}{2}, v = \frac{y}{2}$ and integrating over an appropriate region in the uv plane.

(2 × 15 = 30)