B.Sc. DEGREE (CBCS) EXAMINATION, OCTOBER 2019

Third Semester

CORE COURSE - MM3CRT01 - CALCULUS

(Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model III Computer Science)

2017 Admission Onwards

24282AFB

Maximum Marks: 80

Time: 3 Hours

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Expand a^x by Maclaurin's series.

2. Write the co-ordinates of the centre of curvature of a curve $y = f(x)$ at a point $P(x, y)$

3. what is an oblique asymptotes.

4. Find the envelope of family of straight line y = mx+ a/m. m being the parameter.

5. Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ if $f(x, y) = y^x$

6. Find $\dfrac{dw}{dt}$ if $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$.

7. Explain the absolute maximum of a continuous function at a point (a, b) defined on a bounded region R.

8. The solid lies between planes perpendicular to the X-axis at x=0 and x=4 . The cross-sections perpendicular to X-axis are squares whose diagonals run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$. Find the area of cross section $A(x)$.

9. Find the volume of solid of revolution generated by rotating the region between the Y-axis and graph of the function $y = x$: $0 \le y \le 1$ about Y-axis.

10. Write the equations for finding surface area of revolution about (i) the X-axis (ii) the Y-axis.

11. Evaluate $\displaystyle\iint_R (10 + x^2 + 3y^2)\, dA$ where $R : 0 \le x \le 1 ; 0 \le y \le 2$

12. Evaluate $\displaystyle\int_0^2 \int_0^2 \int_0^2 dz\, dy\, dx$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Obtain Taylor series expansion in powers of h ior $f(x) = \cos(x + h)$

14. Find the radius of curvature of $\frac{x^2}{9} + \frac{y^2}{16} = 2$ at (3,4).

15. Verify that $w_{xy} = w_{yz}$ where $w = x^2 \tan(xy)$.

16. Find all local extreme values and saddle point, if any, of the function
$$f(x,y) = x^3 - y^3 - 2xy + 6.$$

17. Find the volume of the solid generated by revolving the region bounded by the curves and lines $y = x^2$, $y = 2 - x$, $x = 0$ for x \geq 0 about the Y-axis using shell method.

18. Find the length of the curve $y = \int_0^x \tan t \, dt$, $0 \le x \le \pi/6$

19. Sketch the region of integration and calculate
$$\iint_R \frac{\sin x}{x} \, dA$$
where R is the triangle in the XY-plane bounded by the X-axis and the line $y = x$ and

20. Sketch the region bounded by the lines $x = 0$, $y = 2x$ and $y = x$. express the region's area as double integral and evaluate the integral.

21. Evaluate the cylindrical coordinate integral $\int_0^{2\pi} \int_0^{3} \int_{r^2/3}^{\sqrt{18-r^2}} dz\, r\, dr\, d\theta$ $(6 \times 5 = 30)$

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Find the ranges of values x in which the curve y = 3x³ - 40x² +3x -20 are convex upwards or downwards. Also find their pointsof inflection , equation of the inflectional tangents to the curve and show that they lie on a straight line.

23. (a). If $\sin u = \dfrac{x+y}{\sqrt{x}+\sqrt{y}}$. prove that $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y} = \dfrac{1}{2} \tan u$.

(b). Find the maximum and minimum values that the function $f(x,y) = 3x + 4y$ takes on the circle $x^2 + y^2 = 1$

24. (a). Find the volume of the solid that results when the region enclosed by $y = \sqrt{x}$, $y = 0$ and $x = 9$ revolved about the line $x = 9$.

(b). Find the length of the curve $x = \frac{1}{3}(y^2 + 2)^{3/2}$ from $y = 0$ to $y = 1$.

(c). Find the area of the surface generated by revolving the curve $y = \sqrt{x} - \frac{1}{3}x^{3/2}$; $1 \le x \le 3$, about the X-axis.

25. (a). Evaluate $\iint_R e^{x^2 + y^2} \, dA$ where R is the semi circular region bounded by the X-axis and the curve $y = \sqrt{1 - x^2}$.

(b). Find the Jacobian $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$ for the transformation
$u = x + y + z, v = x + y - z, w = x - y + z$.

$(2 \times 15 = 30)$