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Reg. No................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2020

Second Semester

Faculty of Science

Branch I (a) : Mathematics

MTO 2C 06—ABSTRACT ALGEBRA

(2012—2018 Admissions)

Time : Three Hours — Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Find the order of (8, 4, 10) in the group $Z_{12} \times Z_{60} \times Z_{24}$.
2. Show that $f(x) = x^2 + 8x - 2$ is irreducible over Q. Is $f(x)$ irreducible over R.
3. Find the degree and basis for $Q(\sqrt{2}+\sqrt{3})$ over $Q(\sqrt{3})$. Justify.
4. Show that squaring the circle is impossible.
5. Show that no group of order 36 is simple.
6. Find the conjugates of $\sqrt{2}-\sqrt{3}$ over Q.
7. Find the degree over Q of the splitting field over Q of $x^3 - 1$ in $Q[x]$.
8. Is the field $E = Q\left[\sqrt{2}, \sqrt{3}\right]$ is separable over Q. Justify your answer.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Prove that the polynomial $x^2 - 2$ has no zeros in the rational numbers.
10. Let $f(x) \in F(x)$ and degree of $f(x)$ is 2 or 3. Show that $f(x)$ is reducible over F if and only if it has a zero in F.

Turn over

11. Show that the field C of Complex numbers is an algebraically closed field.

12. Let E be an algebraic extension of a field F. Prove that there exist a finite number of elements $\alpha_1, \alpha_2, \ldots\ldots\alpha_n$ in E such that $E = F(\alpha_1, \alpha_2, \ldots\ldots\alpha_n)$ if and only if E is a finite extension of F.

13. Show that for a prime number p, every group G of order p^2 is abelian.

14. Show that the set of all automorphisms of a field E is a group under function composition.

15. Show that every field of characteristic zero is perfect.

16. Prove that if $f(x)$ is irreducible in $F(x)$, then all zeros of $f(x)$ in $\overline{F}$ have the same multiplicity.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Establish division algorithm for $F(x)$.

18. (a) Establish Eisenstein criterion.

(b) State and prove unique factorization theorem for polynomials.

19. State and prove Kronecker's theorem.

20. Show that if E is a finite extension field of a field F, and K is a finite extension field of E, show that K is a finite extension of F and [K : F] = [K : E] [E : F].

21. Establish the conjugation isomorphisms of algebraic field theory.

22. Prove that a field E where $F \leq E \leq \overline{F}$ is a splitting field over F if and only if every automorphisms of $\overline{F}$ lea ng F fixed maps E onto itself and induces an automorphism of E leaving F fixed.

(3 × 5 = 15)