

E 2128

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Reg. No.....

Name.....

**B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2015**

**Second Semester**

Complementary Course—Statistics

**THEORY OF RANDOM VARIABLES**

(Common for Physics, Mathematics and Computer Applications)

[2013 Admission onwards]

Time : Three Hours

Maximum : 80 Marks

**Part A (Short Answer Questions)**

*Answer all questions. 1 mark each.*

1. What is continuous random variable ?
2. Define p.d.f.
3. What is joint probability distribution ?
4. Define expectation of a function of a random variable.
5. What is raw moment ?
6. Define m.g.f.
7. Define Bowley's measure of skewness.
8. Give any *two* examples of partition values.
9. What are the limits of correlation coefficient ?
10. How do you interpret when the coefficient of correlation is zero ?

(10 × 1 = 10)

**Part B (Brief Answer Questions)**

*Answer any eight questions. 2 marks each.*

11. Explain distribution function of a random variable.
12. When will you say two random variables are independent ?
13. Find  $k$  if :

$$f(x) = \begin{cases} k x e^{-x} ; x \geq 0 \\ 0 \text{ elsewhere} \end{cases}$$

is a p.d.f.

Turn over



14. Show that  $E(k) = k$  where  $k$  is a constant.
15. Find the characteristic function of the variable with probability distribution given by :

$$X : 0 \quad 1 \quad 2 \quad 3$$

$$P(x) : \frac{1}{12} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{3}$$

16. Define conditional expectation.
17. What are the uses of moments in skewness and kurtosis ?
18. What is Scatter diagram ?
19. State the principle of least squares.
20. What are the normal equations for fitting a curve of the form  $y = a \cdot e^{bx}$  ?
21. Why is Pearson's coefficient of correlation known as product moment correlation ?
22. What is the relation between correlation coefficient and regression coefficients ?

(8 × 2 = 16)

### Part C (Descriptive/Short Essay Questions)

Answer any six questions. 4 marks each.

23. Two random variables have the following joint probability distribution :

X \ Y	0	1	2
1	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{11}{32}$
2	$\frac{1}{4}$	$\frac{13}{64}$	$\frac{5}{32}$

Find :

- The marginal distributions.
  - $E(X)$  and  $E(Y)$ .
  - $E(XY)$ .
24. Find  $k$  if  $f(xy) = k \cdot x^2 y^3$ ,  $0 < x < 2, 0 < y < 4$  is a joint p.d.f.



25. For a random variable  $X$  m.g.f. is  $\frac{(1-2t)^{-1}}{t}$ . Find its first four central moments.
26. For two variables  $X, Y$  show that  $[\mathbf{E}(XY)]^2 \leq \mathbf{E}(X^2)\mathbf{E}(Y^2)$ . Give any one application of this result.
27. Derive the relation between first four central moments and first four raw moments.
28. Describe the method of fitting a parabola to a given data.
29. Two regression lines are  $7x - 2y - 29 = 0$  and  $5x - 11y + 8 = 0$ . Find the :
- Ratio of variances of  $X, Y$ .
  - Correlation coefficient.
30. Show that change in scale and origin will not change the value of correlation. What about the case of regression coefficients ?
31. Calculate a measure of skewness based on quartiles of the data :

0-14	15-29	30-44	45-59
3	4	15	2

(6 × 4 = 24)

**Part D (Long Essay Type Questions)***Answer any two questions. 15 marks each.*

32. For the distribution :

$$f(x, y) = \begin{cases} k \cdot \frac{x}{y^2}; & 0 < x < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

find (i)  $k$  ; (ii)  $\mathbf{E}(X | Y)$  ; (iii)  $\mathbf{P}(X < 0.5)$  ; (iv)  $\mathbf{P}(Y > 0.25)$ .

33. What are the properties of characteristic function ? Prove any two of them. Find the characteristic function of a variable with p.d.f.

$$f(x) = \begin{cases} e^{-x}; & x \geq 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Turn over



34. The first four moments of a data about 4 are  $-1.5$ ,  $17$ ,  $-30$  and  $108$ . Find the first four moments about (i) zero ; (ii) 6.
35. From the following data it is required estimate the demand when price is 21. Obtain a suitable regression equation. Also find the estimate :

Price	:	18	24	25	20	28	32
Demand:		8	7	6	10	5	4

(2 × 15 = 30)