

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2019**Second Semester**

Complementary Course—Statistics

THEORY OF RANDOM VARIABLES

[Common for Physics, Mathematics and Computer Applications]

(2013—2016 Admissions)

Time : Three Hours

Maximum Marks : 80

Part A (Short Answer Questions)*Answer all questions.**Each question carries 1 mark.*

1. Give two examples of random variables.
2. What are the two types of random variables ?
3. Define probability density function.
4. State the properties of mathematical expectation.
5. How will you derive the raw moments using moment generating function ?
6. State the properties of characteristic function.
7. State the measures of kurtosis.
8. What is Scatter diagram ?
9. What do you mean by curve fitting ?
10. How do you interpret correlation co-efficients ?

(10 × 1 = 10)

Part B (Brief Answer Questions)*Answer any eight questions.**Each question carries 2 marks.*

11. Define distribution function of a random variable and write down its properties.
12. A continuous random variable X has the pdf $f(x) = a + bx, 0 \leq x \leq 1$. If the mean of the distribution is 0.5, find the values of a and b .

Turn over

13. Define joint probability density function and state its properties.
14. Give an example of random variable whose expectation does not exist.
15. State properties of moment generating function.
16. Show that $V(aX + b) = a^2 V(X)$.
17. What are the measures of skewness in terms of moments?
18. What do you mean by absolute moments.
19. How do you interpret correlation co-efficient?
20. Distinguish between simple correlation and rank correlation.
21. What are regression co-efficients.
22. Why there are two regression lines?

(8 × 2 = 16)

Part C (Descriptive / Short Essay Questions)

*Answer any six questions.
Each question carries 4 marks.*

23. Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.
24. Let X be a continuous random variable with pdf $f(x)$. Let $Y = X^2$. Find the pdf and distribution function of Y .
25. Find the constant c such that the function

$$f(x) = cx^2, \quad 0 < x < 3$$

$$= 0, \quad \text{otherwise}$$

is a pdf. Also compute $p(1 < x < 2)$.

26. Define stochastic independence of two random variables in terms of joint distribution and marginal distribution.
27. If the pdf of the random variable X is $f(x) = e^{-|x|}$, $-\infty < x < \infty$. Derive the moment generating function of X .

28. Establish the relation between raw moments and central moments.
29. If $f(x) = ke^{-|x|}$, $-\infty < x < \infty$, is the pdf of a random variable X. Find (i) the value of k ; (ii) mean; (iii) standard deviation; (iv) mgf.
30. Write a note on Scatter diagram.
31. Fit a straight line to the following data :

x	:	1	2	3	4	5
y	:	14	13	4	5	2

(6 × 4 = 24)

Part D (Long Essay Type Questions)

Answer any **two** questions.

Each question carries 15 marks.

32. If $f(x, y) = 2(x + y - 3xy^2)$, $0 \leq x \leq 1$, $0 \leq y \leq 1$. Find the marginal pdf's of X and Y. Are X and Y are independent.
33. Derive Spearman's formula for rank correlation co-efficient.
34. Fit a curve of the form $y = ax^b$ to the following data :
- | | | | | | | | | |
|-----|---|----|----|-----|-----|-----|-----|-----|
| x | : | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| y | : | 87 | 97 | 113 | 129 | 202 | 195 | 193 |
35. The amount of bread (in hundreds of pounds) X, that a certain bakery is able to sell in a day is found to be a numerical valued random phenomenon with a probability density function $f(x)$ given by

$$\begin{aligned}
 f(x) &= ax, 0 \leq x < 5 \\
 &= a(10 - x), 5 \leq x < 10 \\
 &= 0, \text{ elsewhere.}
 \end{aligned}$$

- (i) Determine the value of a .
- (ii) What is the probability that sales tomorrow exceed 500 pounds.
- (iii) Less than 500 pounds.

(2 × 15 = 30)