

QP CODE: 20000642



Reg No :

Name :

MSc DEGREE (CSS) EXAMINATION , NOVEMBER 2020

Second Semester

CORE - ME010201 - ADVANCED ABSTRACT ALGEBRA

M Sc MATHEMATICS, M Sc MATHEMATICS (SF)

2019 Admission Onwards

94DF5558

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

Answer any eight questions.

Weight 1 each.

1. Show that $\alpha \in \mathbb{C}$ is algebraic over \mathbb{Q} where $\alpha = 1 + \sqrt{2}$
2. Let E be an extension field of F . Then prove that \overline{F}_E is a subfield of E .
3. Prove or disprove: Every UFD is a PID.
4. Show that $\mathbb{Z}[i]$ is an integral domain.
5. Define multiplicative norm on an integral domain.
6. State the Conjugation Isomorphism theorem. Give an example for a conjugation isomorphism.
7. Prove that any two algebraic closures of a field F are isomorphic under an isomorphism leaving each element of F fixed.
8. Let E be an extension of a field F . When we say that a polynomial in $F[x]$ splits in E ? Give an example.
9. When an irreducible polynomial is separable over a field? Give an example also.
10. Define the n th cyclotomic extension of a field F . Give an example.

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any six questions.

Weight 2 each.

11. Prove that trisecting the angle is impossible.
12. If F is a field of prime characteristic p with algebraic closure \overline{F} then prove that $x^{p^n} - x$ has





p^n distinct zeros in \overline{F}

13. Define an irreducible element in a PID D . If p is an irreducible in D and p divides the product $a_1 a_2 \dots a_n$ for a_i in D , then prove that $p|a_i$ for atleast one i .
14. For a Euclidean domain with a Euclidean norm v , prove that $v(1)$ is minimal among all $v(a)$ for nonzero a in D , and also prove that u in D is a unit if and only if $v(u) = v(1)$.
15. Prove that the set of all automorphisms of a field E is a group under function composition.
16. Describe all extensions of the identity map of \mathbb{Q} to an isomorphism mapping $\mathbb{Q}(\sqrt[3]{2}, \sqrt{3})$ onto a subfield of $\overline{\mathbb{Q}}$.
17. Prove that a finite separable extension of a field is a simple extension.
18. State the main theorem of Galois Theory.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. a) If E is a finite extension field of a field F and K is a finite extension field of E , then prove that K is a finite extension of F and $[K:F] = [K:E][E:F]$
b) If E is an extension field of F , $\alpha \in E$ is algebraic over F and $\beta \in F(\alpha)$ then prove that $\deg(\beta, F)$ divides $\deg(\alpha, F)$
20. Prove the following.
 - a) Every PID is a UFD.
 - b) If D is a UFD, then for every nonconstant $f(x)$ in $D[x]$, $f(x) = (c)g(x)$, where c belongs to D and $g(x)$ in $D[x]$ is primitive. Also the element c is unique upto a unit factor in D and $g(x)$ is unique upto a unit factor in D .
21. Define splitting field over a field F . Prove that a field E , $F \leq E \leq \overline{F}$, is a splitting field over F if and only if every automorphism of \overline{F} leaving F fixed induces an automorphism of E leaving F fixed.
22. a) Prove that every field of characteristic zero is perfect.
b) Prove that every finite field is perfect.

(2×5=10 weightage)

