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Reg. No................................

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M.Sc. DEGREE (C.S.S.) EXAMINATION, NOVEMBER 2019

Third Semester

Faculty of Science

Branch I (A)—Mathematics

MT 03 C 13—DIFFERENTIAL GEOMETRY

(2012–2018 Admissions)

Time : Three Hours — Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. How will you visualize the graph of a function $f : U \to R$, $U \subset R^2$, given its level sets.
2. Give an example to show that the set of vectors tangent at a point p of a level set might be all of R_p^{n+1}.
3. Define Gauss map and sketch it for 1 – surface of R^2.
4. Prove that in an n-plane, parallel transport is path independent.
5. Find the curvature of $\vec{r} = (\cos 2\mu, \sin 2\mu, 2 \sin \mu)$.
6. Explain normal curvature and oriental n-surface.
7. Explain global property with example.
8. Explain : Curvature of surfaces.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Show that a connected n-surface $S \subset R^{n+1}$ has exactly two smooth unit normal vector fields.
10. State and prove the theorem on the existence and uniqueness of integral curves. Extend it to n-surface.
11. Show that a geodesic can be found to pass through any given point and have a given direction on a surface. Show also that the geodesic is uniquely determined by the initial conditions.
12. Show that parallel transport from p to q along a piecewise smooth parametrized curve is a vector space isomorphism which preserves dot product.

Turn over

13. Find the arc length of one complete turn of the circular helix

$$r(u) = (a \cos u, a \sin u, b u) - \infty < u < \infty.$$

14. Establish necessary and sufficient condition for a global parametrization of an oriented plane curve.

15. Show that parametric equations of a surface need not be unique.

16. Let S be the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$, $(a, b, c \neq 0)$, oriented by its outward normal. Find the Gaussian curvature.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Define covariant derivative of a vector $\overline{X}$. Establish necessary and sufficient conditions for a parametrised curve $\alpha : I \to s$ be a geodesic.

18. (a) Show that the set SL (3) of all 3 × 3 real matrices with determinant equal to 1 is an 8-surface is R^9.

    (b) What is the tangent space to SL (3) at $p = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$?

19. Define Weingarten map and prove it is self adjoint.

20. Let C be a connected oriented plane curve and $\beta : I \to C$ be a unit speed global parametrisation of C. Prove that β is $1 - 1$ or periodic.

21. (a) State and prove inverse function theorem on n-surfaces.

    (b) Differentiate between curvature of plane curves and curvature of surfaces.

22. Establish the two theorems to show that surfaces and parametrised surfaces are the same.

(3 × 5 = 15)