

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2015**Second Semester**

Core Course 2—ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES

(Common for B.Sc. Mathematics Model I, Model II and B.Sc. Computer Applications)

[2013 Admission onwards]

Time : Three Hours

Maximum : 80 Marks

Part A

*Answer all questions.
Each question carries 1 mark.*

1. Define the hyperbolic sine function $\sinh x$.
2. Separate into real and imaginary parts $\tan(x - iy)$.
3. Write the equation of the normal at the point 't' to the parabola $y^2 = 4ax$.
4. What is the condition for the normals at t_1 and t_2 to the parabola intersect at a point on the parabola $y^2 = 4ax$?
5. What is the equation of the normal at the point θ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
6. Find the equation of the asymptotes of $2x^2 + 2xy - 3x + y = 0$.
7. Find the point of intersection of the tangents at t_1 and t_2 on the rectangular hyperbola $xy = c^2$.
8. What is the polar equation of a straight line?
9. What are non-singular matrices?
10. Find the characteristic polynomial of $\begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$.

(10 × 1 = 10)

Part B

*Answer any eight questions.
Each question carries 2 marks.*

11. Show that in a parabola, the subnormal is constant.

Turn over

12. Show that the sum of the ordinates of the feet of the normals from any point to a parabola is zero.
13. If s and s' are the foci of the ellipse and p any point on it, show that $sp + s'p = 2a$.
14. Find the condition for $lx + my + u = 0$ to be a normal to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
15. Find the angle between the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
16. Find the equation of the hyperbola conjugate to $4x^2 + 13xy + 3y^2 + x + 3y - 25 = 0$.
17. What is the polar equation of a circle?
18. Prove that $\log(-1) = i\pi$.
19. If $\tan \frac{x}{2} = \tanh \frac{x}{2}$. Prove that $\cos x \cdot \cosh x = 1$.
20. What are the elementary transformations on a matrix.

21. Reduce to the normal form $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$.

22. Find the eigen values of $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

(8 × 2 = 16)

Part C

*Answer any six questions.
Each question carries 4 marks.*

23. Show that tangents at the ends of a focal chord of a parabola, intersect at right angles on the directrix.
24. Prove that the equation to the locus of the point of intersection of two normals to the parabola $y^2 = 4ax$ which are perpendicular to each other is the curve $y^2 = a(x - 3a)$.
25. Show that the eccentric angles of the ends of a pair conjugate diameters of an ellipse differ by a right angle.

26. In the ellipse $3x^2 + 7y^2 = 21$. Find the equation of the equi conjugate diameters and their lengths.
27. If e and e_1 are the eccentricities of a hyperbola and its conjugate, show that $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$.
28. If $\cos(x + iy) = \cos \theta + i \sin \theta$, show that $\cos 2x + \cosh 2y = 2$.
29. Sum the series $\cos x - \frac{1}{2} \cos 2x + \frac{1}{3} \cos 3x \dots \infty$.
30. Using Cayley-Hamilton theorem, find A^3 if $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$.
31. Find the eigen values and the corresponding eigen vectors to $\begin{bmatrix} 5 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$.

(6 × 4 = 24)

Part D

*Answer any two questions.
Each question carries 15 marks.*

32. (a) If $c = \cos^2 \theta - \frac{1}{3} \cos^3 \theta \cos 3\theta + \frac{1}{5} \cos^5 \theta \cos 5\theta + \dots$ prove that $\tan 2c = 2 \cot^2 \theta$.
- (b) Resolve into real factors $x^8 + 1$.
33. (a) Prove that the locus of the poles of all normal chords of the rectangular hyperbola $xy = c^2$ is the curve $(x^2 - y^2)^2 + 4c^2 xy = 0$.
- (b) Find the equation of a rectangular hyperbola referred to its asymptotes as axes.
34. (a) Find the locus of the foot of the perpendiculars drawn from the pole to the tangents to the circle $r = 2a \cos \theta$.
- (b) Find the equation of the tangent of α to the conic $\frac{l}{r} = 1 + e \cos(\theta - r)$.

Turn over

35. (a) Using matrix method solve :

$$2x - y + 3z = 9$$

$$x + y + z = 6$$

$$x - y + z = 2.$$

(b) Solve the system of equations :

$$x + 2y + 3z = 0$$

$$2x + y + 3z = 0$$

$$3x + 2y + z = 0.$$

(2 × 15 = 30)