E 1685 (Pages : 3) Reg. No..............................

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# B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2015

## Sixth Semester

Core Course—REAL ANALYSIS

(For B.Sc. Mathematics Model I and II and B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

### Part A

*Answer* **all** *questions.*
*Each bunch of four questions has weight* 1.

I. 1 State Cauchy's general principle of convergence of a series.

2 What is a necessary condition for convergence of a series.

3 Show that the series $\sum \frac{1}{n}$ diverges.

4 What is a geometric series ?

II. 5 State Gauss's test.

6 Is every convergent series converges absolutely.

7 When we say that a function has a removable discontinuity at a point.

8 Show that if a function $f$ is continuous at a point $c$, then $|f|$ is also continuous at $c$.

III. 9 Show that the function :

$$f(x) = \begin{cases} x \sin(1/x) & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

is continuous at $x = 0$.

10 State maximum-minimum theorem.

11 State Riemann's criterion for integrability.

12 What do you mean by a monotone function ?

IV. 13 If $|f|$ is integrable, then $f$ is integrable, write True *or* False.

14 State Weierstrass's M test for uniform convergence.

15 Write whether the series $\sum \frac{\cos n\theta}{n^p}$ converges uniformly for $p > 1$.

Turn over

16 Show that the series $\sum \frac{x^n}{n^2}$ converges uniformly in $[-1, 1]$.

(4 × 1 = 4)

## Part B

*Answer any* **five** *questions.*
*Each question has weight* 1.

17 Show that the series $1^2 + 2^2 + 3^2 + \dots\dots\dots$ diverges to $+\infty$.

18 Examine the convergence of the series $\frac{1}{3} + \frac{1 \cdot 2}{3 \cdot 5} + \frac{1 \cdot 2 \cdot 3}{3 \cdot 5 \cdot 7} + \dots\dots\dots$

19 Examine the convergence of the series $\frac{1}{1p} - \frac{1}{2p} + \frac{1}{3p} \dots, p > 0$.

20 Show that the function

$$f(x) = \begin{cases} \frac{\sin 2x}{x}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$$

has a removable discontinuity at the origin.

21 Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $[0, 1]$.

22 Let $f : I \to R$ be a bounded function and $P_1$ and $P_2$ any two partitions on I, show that $L(p_1, f) \leq U(p_2, f)$.

23 Let a function $f$ be defined on $[-1, 1]$ as

$$f(x) = \begin{cases} k, & \text{positive constant if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

show that $f$ is integrable on $[-1, 1]$ and the value of the integral is $2k$.

24 Test for uniform convergence the series $\frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} + \dots, -\frac{1}{2} \leq x \leq \frac{1}{2}$.

(5 × 1 = 5)

## Part C

*Answer any* **four** *questions.*
*Each question has weight* 2.

25 Show that the series

$$\frac{1}{(\log 2)^p}+\frac{1}{(\log 3)^p}+\ldots\ldots+\frac{1}{(\log n)^p}+\ldots\ldots \text{ diverges for } p>0.$$

26 Show that the series $x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots$ converges absolutely for all values of $x$.

27 Show that the function $f(x)$ defined on R by

$$f(x)=\begin{cases} x & \text{when } x \text{ is irrational} \\ -x & \text{when } x \text{ is rational} \end{cases}$$

is continuous only at $x=0$.

28 Show that the function $f$ defined by

$$f(x)=\begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

is not integrable on any interval.

29 Compute $\int_{-1}^{1} f\,dx$, where $f(x)=|x|$.

30 State and prove Abel's test.

(4 × 2 = 8)

## Part D

*Answer any* **two** *questions.*
*Each question has weight* 4.

31 State and prove Leibnitz test.

32 Show that if a function $f$ is continuous on a closed interval $[a, b]$, then it attains its bounds atleast once in $[a, b]$.

33 (i) Show that if $f$ is integrable on $[a, b]$, then $f^2$ is also integrable on $[a, b]$.

(ii) If $f_1$ and $f_2$ are both integrable on $[a, b]$, then show that $f_1 f_2$ is also integrable on $[a, b]$.

(2 × 4 = 8)