QP CODE: 20101175

20101175

Reg No : ....

Name : ....

B.Sc. DEGREE (CBCS) EXAMINATION, NOVEMBER 2020

Second Semester

Core Course - MM2CRT01 - MATHEMATICS - ANALYTIC GEOMETRY, TRIGONOMETRY AND DIFFERENTIAL CALCULUS

(Common for B.Sc Computer Applications Model III Triple Main,B.Sc Mathematics Model I,B.Sc Mathematics Model II Computer Science)

2017 ADMISSION ONWARDS

93288FB8

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

Each question carries 2 marks.

1. Find the locus of the middle points of a system of parallel chords of the parabola $y^2 = 4ax$.
2. Show that for a parabola, the directrix is the polar of the focus.
3. Find the condition that the lines $lx + my + n = 0$ and $l_1x + m_1y + n_1 = 0$ to be conjugate with respect to the parabola $y^2 = 4ax$.
4. Show that the locus of the mid-point of a system of parallel chords of an ellipse is a straight line passing through its centre.
5. Find a polar equation for the circle $x^2 + (y-3)^2 = 9$.
6. Determine the equation for a line in polar coordinates when the line passes through the pole. Also give an example.
7. Prove that $\cos(x - y) = \cos x \cos y + \sin x \sin y$.
8. Prove that $\cosh 2x = \cosh^2 x + \sinh^2 x$.
9. Separate into real and imaginary parts $tanh(\alpha + i\beta)$.
10. Find the n^{th} dervative of $(ax+b)^n$.
11. Find the n^{th} derivative of sinxcos3x.
12. Evaluate $lim_{\theta \to \frac{\pi}{2}} \frac{log(\theta - \frac{\pi}{2})}{tan\theta}$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. If SY and S'Y' be perpendiculars from the foci upon the tangent at any point P of the ellipse, then show that Y and Y' lie on a circle and SY.S'Y' = b^2.

14. Two tangents from a point to the parabola $y^2 = 4ax$ make with each other an angle 45^0. Prove that the locus of their point of intersection is given by $y^2 - 4ax = (x+a)^2$.

15. From the points on the line $2x - 3y + 4 = 0$ tangents are drawn to the parabola $y^2=4ax$. Show that the chord of contact passes through a fixed point.

16. Define equi-conjugate diameters. Derive the combined equation of equi-conjugate diameters of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

17. Show that the tangents at the extremities of any focal chord of a conic intersect on the corresponding directrix.

18. Sum the series $cos\alpha + ccos(\alpha+\beta) + \frac{c^2}{2!}cos(\alpha+2\beta)+.....$where c is less than unity.

19. Sum the series $csin^2\alpha - \frac{1}{2}c^2sin^22\alpha + \frac{1}{3}c^3sin^23\alpha-...$where c is less than unity.

20. If $y = [log\frac{x+\sqrt{x^2-a^2}}{a}]^2 + klog(x+\sqrt{x^2-a^2})$, then prove that $(x^2-a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2a$.

21. Determine $lim(cotx)^{\frac{1}{logx}}, x \to 0$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Find the orthoptic locus of (a) the parabola $y^2 = 4ax$ (b) the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (c) the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

23. A circle passing through the focus of a conic whose latus rectum is $2l$ meets the conic in four points whose distances from the focus are $r_1, r_2, r_3\ and\ r_4$. Prove that $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} = \frac{2}{l}$.

24. Factorize the expression $x^n - 1$

25. (a) If x+y=1, find the n^{th} derivative of x^ny^n.
(b) If $y = e^{asin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$.
(c) If $y=(x^2-1)^n$, prove that $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.

(2×15=30)