

QP CODE: 20101175



20101175

Reg No : .....

Name : .....

**B.Sc. DEGREE (CBCS) EXAMINATION, NOVEMBER 2020**

**Second Semester**

**Core Course - MM2CRT01 - MATHEMATICS - ANALYTIC GEOMETRY,  
TRIGONOMETRY AND DIFFERENTIAL CALCULUS**

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc  
Mathematics Model II Computer Science)

2017 ADMISSION ONWARDS

93288FB8

Time: 3 Hours

Max. Marks : 80

**Part A**

*Answer any ten questions.*

*Each question carries 2 marks.*

1. Find the locus of the middle points of a system of parallel chords of the parabola  $y^2 = 4ax$ .
2. Show that for a parabola, the directrix is the polar of the focus.
3. Find the condition that the lines  $lx + my + n = 0$  and  $l_1x + m_1y + n_1 = 0$  to be conjugate with respect to the parabola  $y^2 = 4ax$ .
4. Show that the locus of the mid-point of a system of parallel chords of an ellipse is a straight line passing through its centre.
5. Find a polar equation for the circle  $x^2 + (y - 3)^2 = 9$ .
6. Determine the equation for a line in polar coordinates when the line passes through the pole. Also give an example.
7. Prove that  $\cos(x - y) = \cos x \cos y + \sin x \sin y$ .
8. Prove that  $\cosh 2x = \cosh^2 x + \sinh^2 x$ .
9. Separate into real and imaginary parts  $\tanh(\alpha + i\beta)$ .
10. Find the  $n^{\text{th}}$  derivative of  $(ax + b)^n$ .
11. Find the  $n^{\text{th}}$  derivative of  $\sin x \cos 3x$ .
12. Evaluate  $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\log(\theta - \frac{\pi}{2})}{\tan \theta}$ .

(10×2=20)





**Part B**

Answer any six questions.

Each question carries 5 marks.

13. If SY and S'Y' be perpendiculars from the foci upon the tangent at any point P of the ellipse, then show that Y and Y' lie on a circle and  $SY.S'Y' = b^2$ .
14. Two tangents from a point to the parabola  $y^2 = 4ax$  make with each other an angle  $45^\circ$ . Prove that the locus of their point of intersection is given by  $y^2 - 4ax = (x+a)^2$ .
15. From the points on the line  $2x - 3y + 4 = 0$  tangents are drawn to the parabola  $y^2=4ax$ . Show that the chord of contact passes through a fixed point.
16. Define equi-conjugate diameters. Derive the combined equation of equi-conjugate diameters of an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .
17. Show that the tangents at the extremities of any focal chord of a conic intersect on the corresponding directrix.
18. Sum the series  $\cos\alpha + c\cos(\alpha + \beta) + \frac{c^2}{2!}\cos(\alpha + 2\beta) + \dots$  where c is less than unity.
19. Sum the series  $c\sin^2\alpha - \frac{1}{2}c^2\sin^22\alpha + \frac{1}{3}c^3\sin^23\alpha - \dots$  where c is less than unity.
20. If  $y = \left[\log \frac{x+\sqrt{x^2-a^2}}{a}\right]^2 + k\log(x + \sqrt{x^2 - a^2})$ , then prove that  $(x^2 - a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 2a$ .
21. Determine  $\lim(\cot x)^{\frac{1}{\log x}}, x \rightarrow 0$ .

(6×5=30)

**Part C**

Answer any two questions.

Each question carries 15 marks.

22. Find the orthoptic locus of (a) the parabola  $y^2 = 4ax$  (b) the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  (c) the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ .
23. A circle passing through the focus of a conic whose latus rectum is  $2l$  meets the conic in four points whose distances from the focus are  $r_1, r_2, r_3$  and  $r_4$ . Prove that  $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} = \frac{2}{l}$ .
24. Factorize the expression  $x^n - 1$
25. (a) If  $x+y=1$ , find the  $n^{\text{th}}$  derivative of  $x^n y^n$ .  
 (b) If  $y = e^{a\sin^{-1}x}$ , prove that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$ .  
 (c) If  $y=(x^2 - 1)^n$ , prove that  $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$ .

(2×15=30)

