

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2016**Fifth Semester****Core Course—DIFFERENTIAL EQUATIONS**

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

(2013 Admission onwards)

Time : Three Hours

Maximum : 80 Marks

Part A

*Answer all questions from this part.
Each question carries 1 mark.*

1. What is the order of the differential equation $y \left(\frac{dy}{dx} \right)^2 + 2x \frac{dy}{dx} - y = 0$.
2. Find the differential equation corresponding to the primitive $y = 2ax + bx^2$.
3. What is an exact differential equation ?
4. Write the general form of a linear differential equation with constant co-efficients.
5. Solve the equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.
6. What is a singular point ?
7. What is Bessel function of the first kind of order p .
8. Write the general form of Bessel's equation.
9. Write a parametric equation of the surface $x^2 + y^2 + z^2 = a^2$.
10. What do you mean by a complete integral of a partial differential equation ?

(10 × 1 = 10)

Part B

*Answer any eight questions.
Each question carries 2 marks.*

11. Solve the equation $\frac{dy}{dx} + \frac{x}{y} = 0$.
12. Solve $x(x-y) \frac{dy}{dx} + y^2 = 0$.

Turn over

13. Solve $\frac{dy}{dx} + \frac{y}{x} = x^2$, given that when $x = 1, y = 1$.
14. Solve $x \frac{dy}{dx} + y = x^2 y^2 \log x$.
15. Solve $\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = e^x$.
16. Solve $\frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$.
17. Solve $(D^2 + D + 2)y = x^2$.
18. Solve $\frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} - y = 0$ in powers of x .
19. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
20. Prove that $\frac{d}{dx} \{x^p J_p(x)\} = x^p J_{p-1}(x)$.
21. Derive a partial differential equation by eliminating f from $z = e^y f(x + y)$.
22. Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1, x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$.

(8 × 2 = 16)

Part C

*Answer any six questions.
Each question carries 4 marks.*

23. Show that $(e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0$ is exact and hence solve it.
24. Find the integrating factor of the differential equation $(y - xy^2) \, dx - (x + x^2 y) \, dy = 0$ and hence solve it.
25. Solve the equation $(D^2 - 2D + 1)y = x \sin x$.

26. Solve $(D^2 - 4D + 3)y = e^{-x} \sin x$.
27. Prove that $\frac{d}{dx} \{x J_p(x) J_{p+1}(x)\} = x \{J_p^2(x) - J_{p+1}^2(x)\}$.
28. Given that $y_1 = x^2$ is one solution of $xy'' + xy' - 4y = 0$. Find the general solution.
29. The equation $x^2 y'' - 3xy' + (4x + 4)y = 0$ has only one Frobenius series solution. Find it.
30. Find the integral curves of the equation $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$.
31. Find the general solution of $(1+x^2)y'' + 2xy' - 2y = 0$ in terms of power series in x .

(6 × 4 = 24)

Part D

*Answer any two questions.
Each question carries 15 marks.*

32. (a) Find a particular solution of $y'' + 2y' + 5y = e^{-x} \sec 2x$ by the method of variation of parameters.
- (b) Find a particular solution of $y'' - 2y' + y = 2x$ by the method of undetermined co-efficients.
33. (a) Find the general solution of $x^2 y'' + 3xy' + 10y = 0$.
- (b) Solve $D^2 y + \left(1 + \frac{2}{x} \cot x - \frac{2}{x^2}\right) y = x \cos x$ given that $\frac{\sin x}{x}$ is a complimentary function.
34. Obtain the series solution of the equation $x(1-x) \frac{d^2 y}{dx^2} - (1+3x) \frac{dy}{dx} - y = 0$.
35. Find the general integral of the linear partial differential equation :
- $$px(x+y) = qy(x+y) - (x-y)(2x+2y+z).$$

(2 × 15 = 30)