18002223

18002223

Reg. No.................................

Name.....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, DECEMBER 2018

First Semester

Faculty of Science

Branch I (a) : Mathematics

MT01C03—MEASURE THEORY AND INTEGRATION

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. Define Lebesgue outer measure $m^*(A)$ of a subset A of $\mathbb{R}$.

2. Give an example of a continuous function g and a measurable function h such that $h \circ g$ is not measurable.

3. Show that if f is integrable over E, so is $|f|$. Does the integrability of $|f|$ imply that of f. Justify.

4. State Vitali lemma.

5. State Lebesgue dominated convergence theorem.

6. Show that linear combination of two measures v_1, v_2 that are absolutely continuous with respect to μ is absolutely continuous.

7. State Fubini's theorem.

8. If $f_n \to f$ in measure. Show that there is a subsequence $\{f_{n_k}\}$ which converges to f a.e.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. Show that every Borel set is measurable.

10. Show that the outer measure is translation invariant.

11. Let $< f_n >$ be a sequence of integrable functions such that $f_n \to f$ a.e. with f integrable. Prove that $\int |f - f_n| \to 0$ if and only if $\int |f_n| \to \int |f|$.

Turn over

12. Let $< f_n >$ be a sequence of non-negative measurable functions that converge to f, and suppose $f_n \le f$ for each n. Prove that $\int f = \lim \int f$.

13. Show that if f is integrable with respect to μ, then for a given $\in > 0$ there is a simple function φ such that $\int |f - \varphi| \, d\mu < \in$.

14. Prove that Lebesgue decomposition is unique.

15. Show that if a sequence of measurable functions converges in measure, then the limit function is unique a.e.

16. By integrating $\dfrac{e^{-y} \sin 2xy}{y}$, show that $\int_0^\infty \dfrac{e^{-y}\left(\sin^2 y\right)}{y} = \dfrac{1}{4} \log 5$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. Suppose $< f_n >$ be a sequence of measurable functions. Prove that $\overline{\lim} f_n$ and $\underline{\lim} f_n$ are measurable.

18. State and prove Monotone convergence theorem.

19. Show that there is a sequence of non-negative simple functions (each of which vanishes outside a set of finite measure) that converges to a non-negative measurable function.

20. State and prove Radon-Nikodym theorem.

21. Let $f_n \to f$ a. e. Suppose that $|f_n| \le g$ (an integrable function) prove that $f_n \to f$ a.u.

22. State and prove Fubini's theorem.

(3 × 5 = 15)