QP CODE: 20000642

20000642

Reg No : ....

Name : ....

MSc DEGREE (CSS) EXAMINATION , NOVEMBER 2020

Second Semester

CORE - ME010201 - ADVANCED ABSTRACT ALGEBRA

M Sc MATHEMATICS, M Sc MATHEMATICS (SF)

2019 Admission Onwards

94DF5558

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Show that* $\alpha \in \mathbb{C}$ *is algebraic over* $\mathbb{Q}$ *where* $\alpha = 1 + \sqrt{2}$
2. *Let* E *be an extension field of* F. *Then prove that* $\overline{F}_E$ *is a subfield of* E.
3. Prove or disprove: Every UFD is a PID.
4. Show that $\mathbb{Z}[i]$ is an integral domain.
5. Define multiplicative norm on an integral domain.
6. *State the Conjugation Isomorphism theorem. Give an example for a conjugation isomorphism.*
7. *Prove that any two algebraic closures of a field* F *are isomorphic under an isomorphism leaving each element of* F *fixed.*
8. *Let* E *be an extension of a field* F. *When we say that a polynomial in* $F[x]$ *splits in* E? *Give an example.*
9. *When an irreducible polynomial is separable over a field? Give an example also.*
10. *Define the* n*th cyclotomic extension of a field* F. *Give an example.*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Prove that trisecting the angle is impossible.*
12. *If* F *is a field of prime characteristic* p *with algebraic closure* $\overline{F}$ *then prove that* $x^{p^n} - x$ *has*

p^n distinct zeros in $\overline{F}$

13. Define an irreducible element in a PID D. If p is an irreducible in D and p divides the product $a_1 a_2 \ldots a_n$ for a_i in D, then prove that $p|a_i$ for atleast one i.

14. For a Euclidean domain with a Euclidean norm v, prove that v(1) is minimal among all v(a) for nonzero a in D, and also prove that u in D is a unit if and only if v(a) = v(1).

15. *Prove that the set of all automorphisms of a field E is a group under function composition.*

16. *Describe all extensions of the identity map of $\mathbb{Q}$ to an isomorphism mapping $\mathbb{Q}(\sqrt[3]{2}, \sqrt{3})$ onto a subfield of $\overline{Q}$.*

17. *Prove that a finite separable extension of a field is a simple extension.*

18. *State the main theorem of Galois Theory.*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) If E is a finite extension field of a field F and K is a finite extension field of E, then prove that K is a finite extension of F and [K:F]=[K:E][E:F]

    b) If E is an extension field of F, $\alpha \in E$ is algebraic over F and $\beta \in F(\alpha)$ then prove that $deg(\beta, F)$ divides $deg(\alpha, F)$

20. Prove the following.

    a) Every PID is a UFD.

    b) If D is a UFD, then for every nonconstant f(x) in D[x], f(x) = (c)g(x), where c belongs to D and g(x) in D[x] is primitive. Also the element c is unique upto a unit factor in D and g(x) is unique upto a unit factor in D.

21. *Define splitting field over a field F. Prove that a field E, $F \le E \le \overline{F}$, is a splitting field over F if and only if every automorphism of $\overline{F}$ leaving F fixed induces an automorphism of E leaving F fixed.*

22. a) Prove that every field of characteristic zero is perfect.

    b) Prove that every finite field is perfect.

(2×5=10 weightage)