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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2015

Fourth Semester

Complementary Course—Statistics—STATISTICAL INFERENCE

(Common for B.Sc. Mathematics Model I Physics, and Model I B.Sc. Computer Applications)

[2013 Admissions]

Time : Three Hours Maximum : 80 Marks

Use of scientific calculator and statistical tables are permitted.

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. If X~ Poisson (λ), given an unbiased estimator for λ.
2. In 10,000 tosses of a coin tail turns up 4950 times. Is it reasonable to think that coin is unbiased ?
3. Give two methods of point estimation.
4. Give two criteria for point estimation.
5. Distinguish between null and alternative hypothesis.
6. What are type-I and type-II errors ?
7. What are level of significance and power of a test ?
8. What is the null hypothesis is one-way ANOVA ?
9. What is the criteria for a large sample test in terms of the sample size ?
10. State the Cramer-Rao inequality.

(10 × 1 = 10)

Part B

Answer any **eight** *question.*
Each question carries 2 *marks.*

11. Compare and contrast *t*-test and ANOVA.
12. If 13. 3, 14. 7, 10. 2, 8. 3, 11. 7, 17. 6, 10. 8, and 18. 8 is a sample from the population X~U (a, b), write down an estimate each for the parameters a and b.
13. Distinguish between and estimator and an estimate with an example each.
14. How is an internal estimator different from a point estimator ?

Turn over

15. Obtain a sufficient estimator for λ if X~ Poisson (λ).

16. What will be the sample correlation coefficient computed from two bivariate observations ?

17. Show that in estimating the mean of a normal population, sample mean is more efficient than the sample median.

18. Samples of size 2 are drawn from a population with values 14, 10, 5, 17, 9, 16, 20, 15. Suggest an unbiased estimate of the population mean and obtain its variance.

19. Based on independent observations on X~ Poisson (λ), obtain the MLEs of λ and $e^{-\lambda}$.

20. A consumer wants to estimate the proportion of defective products from a factory by taking a sample from it. Determine the sample size if the estimate should not differ from the value by more than 0.03 with probability not less than 0.95.

21. Define F-distribution. If $X \sim F_{m, n}$, what is the distribution of 1/X ? If $Y \sim t_n$, what is the distribution of Y^2 ?

22. In testing H_0 : An observation belongs to N (25, 4) against H_1 : the observation belongs to N (30, 4), the critical region is set up as $X \geq 29$, where X is the observation. Calculate the probabilities of the two kinds of errors.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. If $x_1, x_2, \ldots, x_n$ is a random sample from a population with variance σ^2, show that $\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is an unbiased estimator for σ^2.

24. Give an example each of an estimator that is (i) both unbiased and consistent ; (ii) not unbiased but consistent.

25. Construct a 95% confidence interval for σ^2 if you are given a sample of size 10 having variance 25 from X~ N (μ, σ^2).

26. Describe the method of moment estimator.

27. Find E(X) if X~ beta (θ, 1) and also the method of moments estimator for θ.

28. Find the best critical region for testing $H_0 : \theta = 2$ against $H_1 : \theta = 4$ using a sample of size m from N (θ, 1).

29. Obtain the MLE of θ in the p.d.f, $f(x, \theta) = (1+\theta)\, x^2, 0 < x < 1$, based on a sample of size n. Examine whether the estimate is sufficient for θ.

30. Obtain a sufficient estimator for μ and σ in $X \sim N(\mu, \sigma^2)$.

31. Derive a test procedure for testing $H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$ where μ is the mean of a normal population.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. The following table shows the number of defective devices found in 200 samples of size 4. Calculate the proportion of defective devices. Assuming binomial distribution to hold, use this value to calculate the theoretical distribution of defective and compare this with the original figure and test for its goodness of fit.

Number of defectives	...	0	1	2	3	4
Number of samples	...	65	84	40	10	1

33. Calculate the correlation coefficient between X and Y in the following data and test its significance.

x	...	24	30	33	35	36	36	37	37	38	40	43	49
y	...	41	39	47	51	43	40	57	46	50	59	61	52

34. Two sample polls of votes for two candidates A and B are taken, one each from among the residents of rural and urban areas. The results are given in the following table. Examine whether the type of area in related to voting preferences.

Area	Votes for A	Votes for B	Total
Rural	620	380	1000
Urban	550	450	1000
Total	1170	830	2000

35. A drug manufacturing company wanted to know about the length of time their product retained its potency. A random sample each from the production line and those stored for a period of one year were taken and analysed for potency. The observations obtained from each sample are given below. Construct a 95% confidence interval for $\mu_1 - \mu_2$, where μ_1 is the mean potency of the sample from the production line and μ_2 is the mean potency of the sample from those stored for year.

From the production time ...	10.2,	10.6,	10.5,	10.7,	10.3,	10.2,	10.8,	10.0,	9.8,	10.6
From those stored for 1 year...	9.8,	9.7,	9.6,	9.5,	10.1,	9.6,	10.2,	9.8,	10.1,	9. 9.

Assume equality of variances and normality for both populations.

(2 × 15 = 30)