QP CODE: 22102150

22102150

Reg No : .....

Name : .....

**B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, JULY 2022**

**First Semester**

**Complementary Course - MM1CMT01 - MATHEMATICS - PARTIAL DIFFERENTIATION, MATRICES, TRIGONOMETRY AND NUMERICAL METHODS**

(Common to B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III , B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance)

2017 Admission Onwards

5F8F1A7B

Time: 3 Hours Max. Marks : 80

**Part A**

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the value of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(4, -5)$ if $f(x, y) = x^2 + 3xy + y - 1$.
2. Find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ if $f(x, y) = x + y + xy$.
3. Find $\frac{dy}{dx}$ if $y^2 - x^2 - \sin xy = 0$.
4. Define singular matrix.
5. Reduce the matrix $A = \begin{bmatrix} 3 & 6 \\ 4 & 3 \end{bmatrix}$ into its normal form.
6. Define characteristic vector of a square matrix.
7. Prove that $\cos 4\theta = \cos^4 \theta - 6 \cos^2 \theta \sin^2 \theta + \sin^4 \theta$.
8. If $x = \cos \theta + i \sin \theta$, find $x^4 + \frac{1}{x^4}$ and $x^4 - \frac{1}{x^4}$.
9. If $x$ is real, show that $\sinh^{-1} x = \log(x + \sqrt{x^2 + 1})$.
10. Separate $\cosh(\alpha + i\beta)$ into real and imaginary parts.
11. Give the first and second approximations to the root of an equation $f(x) = 0$ using the method of false position.
12. Use the iteration method to find, correct to three decimal places, a real root of the equation $e^{-x} = 10x$. Take the first approximation as $x_0 = 0$.

(10×2=20)

**Part B**

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Sketch the domain of the function $f(x,y)=\frac{1}{\sqrt{16-x^2-y^2}}$. Identify whether it is bounded or unbounded, open or closed?

14. Verify whether the function $f(x,y)=\ln\sqrt{x^2+y^2}$ satisfies the two-dimensional Laplace equation $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$.

15. Use chain rule to evaluate $\frac{dw}{dt}$ at $t=1$ if $w=z-\sin(xy),\ x=t,\ y=\ln t,\ z=e^{t-1}$.

16. Show that the system of equations $2x-y+2z=8, 3x+2y-2z=-1, 5x+3y-3z=3$ are consistent.

17. If $A$ is non-singular , prove that the characteristic values of $A^{-1}$ are the reciprocals of the characteristic values of $A$.

18. Sum to infinity the series $\frac{1}{2}\sin\theta+\frac{1.3}{2.4}\sin 2\theta+\frac{1.3.5}{2.4.6}\sin 3\theta+\ldots$

19. Sum to infinity the series $1+\frac{\cos\theta}{1!}+\frac{\cos 2\theta}{2!}+\frac{\cos 3\theta}{3!}+\ldots$

20. Obtain a root, correct to three decimal places, for the function $x^3-3x-5=0$ using the bisection method.

21. Use the generalized Newton's method to find a double root of the equation $f(x)=x^3-x^2-x+1=0$ near $1$.

(6×5=30)

**Part C**

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Solve completely system of equations
$x-2y+z-w=0, x+y-2z+3w=0, 4x+y-5z+8w=0, 5x-7y+2z-w=0$

23. Show that the matrix $A=\begin{pmatrix}1&2&2\\2&1&2\\2&2&1\end{pmatrix}$ satisfy Cayley - Hamilton theorem and hence find its inverse.

24. (a) Prove that $\tanh(x+y)=\frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$.
(b) Sum to infinity the series $1+c\cos\alpha+c^2\cos 2\alpha+\ldots$ where $|c|<1$.

25. Use Newton - Raphson method to obtain all the roots, each correct to four decimal places, of the equation $x^3+3x^2-3=0$.
(Hint: Use the initial approximations as $1, -1$ and $-3$.)

(2×15=30)