QP CODE: 19101370 9

19101370

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) EXAMINATION, MAY 2019

Fourth Semester

Complemetary Course - **MM4CMT01 - MATHEMATICS - FOURIER SERIES, LAPLACE TRANSFORM AND COMPLEX ANALYSIS**

(Common for B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology Model I,B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Chemistry Model III Petrochemicals, B.Sc Physics Model III Electronic Equipment Maintenance, B.Sc Geology and Water Management Model III)

2017 Admission onwards

2CBD1F17

Maximum Marks: 80 **Time: 3 Hours**

Part A

Answer any **ten** questions.

Each question carries **2** marks.

1. Prove that sum of even functions are even?
2. Define power series and its center. If the centere is zero, write the power series?
3. Evaluate $\mathscr{L}(t^2)$
4. Find $\mathscr{L}^{-1}\{\frac{1}{s^2+4s}\}$
5. Write $\mathscr{L}\{t^3 f(t)\}$ using a derivative of $\mathscr{L}\{f(t)\}$
6. Find the real and imaginary parts of $z_1 z_2$ where $z_1 = 8 - 3i$ and $z_2 = 9 + 2i$.
7. Find the real and imaginary parts of $\frac{1}{z}$ where $z = 4 - 5i$.
8. Represent $z = -2 + 2i$ in the polar form.
9. Define the natural logarithm $ln(z)$ of a complex number z.
10. State true or false: If f(z) is analytic in a simply connected domain D, then the integral of f(z) is independent of path in D.
11. State Cauchy's integral formula for a doubly connected region.
12. State Morera's Theorem.

(10×2=20)

Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Find the Fourier series expansion $f(x) = x - x^2, -\pi < x < \pi$ and deduce the value of $\frac{\pi^2}{12}$
14. Find the Fourier cosine series of $f(x) = \sin x, x \in [0, \pi]$
15. Evaluate $\mathscr{L}^{-1}\{\frac{4}{(s+1)(s+2)}\}$

Turn Over

16 Find $\mathcal{L}(\cos at)$ and $\mathcal{L}(\sin at)$ using linearity property of Laplace Transforms

17 Find and plot all the cube roots of $8i$.

18. Check the analyticity of the function $f(z) = z\bar{z}$

19. Find the real and imaginary parts of the function $sin(\pi i)$.

20. Find the parametric representation z=z(t) of the line segments with the following end points:

a) z = 0 and z = 1+2i.

b) z = 1-i and z = 9-5i

21. Find an upper bound for the absolute value of the integral $\int_C \frac{dz}{z^2-1}$, where C is the arc of the circle |z| = 2 from z =2 to z = 2i, that lies in the first quadrant.

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22. Derive the Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}[(x^2-1)^n]$ and find the expressions of Legendre polynomials upto degree 5 from this formula

23. Solve the differential equations using Laplace Transforms

(a) $y'' + 2y' + y = e^{-t}$ with $y(0) = -1$ and $y'(0) = 1$

(b) $y'' + a^2y = 0$ given $y(0) = A$ and $y'(0) = B$

24. Check whether the function $u = x^2 + y^2$ is harmonic or not. If YES, find a corresponding analytic function $f(z)$.

25. Integrate the function $\frac{z^2}{(2z-1)^3}$ in counter clockwise sense around the circle |z| =1.

(2×15=30)