

34. (a) Assume that "for all positive integers n , if n is greater than 4, then n^2 is less than 2^n is true. Use universal modus ponens to show that $100^2 < 2^{100}$.
- (b) Prove the contradiction that if $3n + 2$ is odd then n is odd.
- (c) Use proof by cases to show that $n^2 \geq n$ for every integer n .
35. (a) Prove that every composite number can be expressed as the product of prime factors.
- (b) If n is a positive integer and a is prime to n , prove that $a^{\phi(n)} \equiv 1 \pmod{n}$.
- (c) Evaluate $\phi(5040)$.

(2 × 15 = 30)