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Reg. No................................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, OCTOBER 2019

First Semester

Faculty of Science

Branch I (a) : Mathematics

MT 01C02—BASIC TOPOLOGY

(2012—2018 Admissions)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Define Topology on a non-empty set X. Show that intersection of two topologies is again a topology on X.
2. Prove that interior of a set is the same as the complement of the closure of the complement of the set.
3. Define homeomorphism. Does the projection map $\pi_1 : \mathbb{R}^2 \to \mathbb{R}$ defined by $\pi_1(x, y) = x$, closed ? Justify.
4. Prove that composition of two quotient maps is again a quotient map.
5. Prove that continuous function preserves pathconnectedness.
6. Show that every nonempty connected subset is contained in a unique component.
7. Show that normality is a weakly hereditary property.
8. Prove that every subset a topological space X is closed if and only if the space X is a T_1 space.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Prove that a discrete space is second countable if and only if the underlying set is countable.
10. Prove that in a metric space X, a point y is in the closure of a set A if and only if there exists a sequence $\{x_n\}$ such that $x_n \in A$ for all n and $\{x_n\}$ converges to y in X.
11. Prove that every closed surjective map is a quotient map.

Turn over

12. Show that every second countable space is Lindeloff.

13. Prove that topological product of any finite number of connected spaces is connected.

14. Show that a subset of $\mathbb{R}$ is disconnected if and only if it is not an interval.

15. Prove that a continuous bijection from a compact space onto a Hasdorff space is a homeomorphism.

16. Prove that regularity is a hereditary property.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Describe subspace toplogy with suitable examples, and prove the following, let $A \subset X$ where X is a topological space. If $E \subset A$, then $Cl_A(E) = A \cap Cl_X(E)$.

    (b) Show that $\bar{A} = A \cup A'$ for any subset A of a topological space X.

18. (a) Suppose X is a compact topological space, and $A \subset X$ is closed in X. Prove that A is also compact in its relative topology.

    (b) Prove that the product topology is the weak toplogy determined by the projection functions.

19. Prove that every closed and bounded interval is compact.

20. (a) Prove that X is locally connected if and only if X has a base consisting of connected subsets.

    (b) Show that every continuous real valued function on a compact space is bounded and it attains its extrema.

21. Show that every regular Lindeloff space is normal.

22. (a) Show that every continuous, injection from a compact space into a Hausdorff space is an embedding.

    (b) Prove that a compact subset in a Hausdorff space is closed.

(3 × 5 = 15)