19002715

19002715

Reg. No...............................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, OCTOBER 2019

First Semester

Faculty of Science

Branch I (a)—Mathematics

MT01C01—LINEAR ALGEBRA

(2012—2018 Admissions)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*

Each question carries weight 1.

1. Are the vectors (1, 1, 2, 4), (2, – 1, – 5, 2), (1, – 1, – 4, 0), (2, 1, 1, 6), linealy independent in $\mathbb{R}^4$?

2. Find three vectors in $\mathbb{R}^3$ which are linearly independent, and one point lies on the two dimensional subspace.

3. Let V be the vector space of all $n \times n$ matrices over the field F, and let B be a fixed $n \times n$ matrix. Is $T : V \to V$ defined by T (A) = AB – BA, a linear map ?

4. Let T be the unique linear operator on $\mathbb{C}^3$ for which $T \in_1 = (1, 0, i)$ $T \in_2 = (0, 1, 1)$ and $T \in_3 = (i, 1, 0)$., Is T invertible ?

5. Define n – linear map. Give an example.

6. Let A be a 2 × 2 matrix over a field. Prove that det (I + A) = 1 + det A if and only if trace (A) = 0.

7. Show that similar matrices have the same characteristic polynomial.

8. Find a projection E which projects $\mathbb{R}^2$ onto the sub-space spanned by (1, – 1) along the subspace spanned by (1, 2).

(5 × 1 = 5)

Turn over

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. In $\mathbb{C}^3$, show that $\{(1, 0, -i), (1+i, 1-i, 1), (i, i, i)\}$ form a basis, and find the co-ordinates of the vector (a, b, c) in this basis.

10. Let V be a finite-dimensional vector space over the field F, and let W be a subspace of V. Prove that $\dim W + \dim W^0 = \dim V$.

11. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$. Find the matrix-of T in the ordered basis $\{(1, 0, 1), (-1, 2, 1), (2, 1, 1)\}$.

12. If W is a k– dimensional subspace of an n–dimensional vectorspace V, then prove that W is the intersection of $(n-k)$ hyperspaces in V.

13. Prove that the determinant of triangular matrix is the product of its diagonal entries.

14. Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ Prove that T is diagonalizable.

15. State and prove Cayley - Hamilton theorem.

16. Suppose that $f = (x - c_1)^{d_1} \cdots (x - c_k)^{d_k}$ is the characteristic polynomial of an $n \times n$ matrix A. Show that trace $A = c_1 d_1 + c_2 d_2 + \dots + c_k d_k$.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. Let V be a vector space over a subfield of complex numbers. Suppose α, β and γ are linearly independent vectors in V. Prove that $(\alpha + \beta), (\beta + \gamma),$ and $(\gamma + \alpha)$ are linearly independent.

18. If A and B are $n \times n$ complex matrices, show that $AB - BA = I$ is not possible.

19. Consider the linear operator $T(x_1, x_2) = (-x_2, x_1)$, prove that $(T - cI)$ is invertible for every real number c.

20. Let K be a commutative ring with identity and let n be a positive integer. Show that there is precisely one determinant function on the set of $n \times n$ matrices over K.

21. Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Prove that T is diagonalizable if and only if the minimal polynomial for T is of the form $p = (x - c_1) \ldots (x - c_k)$, where $c_1, \ldots, c_k$ are distinct elements of F.

22. Explain the Direct-sum decomposition of a finite dimensional vector space.

(3 × 5 = 15)