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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, OCTOBER 2019

First Semester

Faculty of Science

Branch I (a) : Mathematics

MT 01 C03—MEASURE THEORY AND INTEGRATION

(2012—2018 Admissions)

Time : Three Hours — Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Show that translates of measurable sets are measurable.
2. Define measurable function. Show that sum of two measurable functions is again measurable.
3. State Vitali lemma.
4. Suppose φ, ψ are simple functions which vanish outside a set of finite measure. Prove that $\int (a\varphi + b\psi) = a\int\varphi + b\int\psi$.
5. State Lebesgue dominated convergence theorem.
6. State Radon-Nikodym theorem.
7. Define almost uniform convergence with an example
8. Let μ and ν be complete measures. Show that $\mu \times \nu$ need not be complete.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Suppose f is a measurable real-valued function and g a continuous function defined on $(-\infty, \infty)$. Show that $g \circ f$ is measurable.
10. Prove that the composition of two Borel measurable functions is again Borel measurable.

Turn over

11. Show that $R\overline{\int_a^b} f(x)\,dx = b-a$ and $R\underline{\int_a^b} f(x)\,dx = 0$ for the function

$$f(x)=\begin{cases}0, & x \text{ is irrational}\\ 1, & x \text{ is rational}\end{cases}.$$

12. Let $< f_n >$ be a sequence of integrable functions such that $f_n \to f$ a.e with f integrable. Prove that $\int | f - f_n | \to 0$ if and only if $\int | f_n | \to \int | f |$.

13. Show that if f is integrable with respect to μ, then for a given $\in > 0$ there is a simple function φ such that $\int | f - \varphi | \, d\mu < \in$.

14. Give an example to show that Hahn decomposition need not be unique.

15. Show that $f_n \to f$ in measure, if $f_n \to f$ a.u

16. If $f_n \to f$ in measure, then prove that $| f_n | \to | f |$ in measure.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Show that there exist Non measurable sets.

18. State and prove Monotone convergence theorem.

19. State and prove bounded convergence theorem.

20. Explain Caratheodory extension of measures

21. Let $f_n \to f$ a.e. Suppose that $| f_n | \le g$ (an integrable function) prove that $f_n \to f$ a.u

22. State and prove Fubini's theorem.

(3 × 5 = 15)