

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2015**First Semester**

Core Course—FOUNDATION OF MATHEMATICS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

[2013 Admission onwards]

Time : Three Hours

Maximum : 80 Marks

Part A (Short Answer Questions)*Answer all questions.**Each question carries 1 mark.*

1. Draw a Venn diagram representing $\bar{A} \cap \bar{B}$.
2. Define the floor function.
3. When is a relation on a set said to be antisymmetric ?
4. Represent the relation $\{(1,1), (1,2), (1,3)\}$ on the set $\{1, 2, 3\}$ with a matrix (with elements of the set listed in increasing order).
5. Define equivalence relation on a set.
6. Write the inverse of the conditional statement "If it snows tonight, then I will stay at home".
7. What do you mean by an exhaustive proof ?
8. State the fundamental theorem of arithmetic.
9. Find the number of divisors of 2000, and their sum.
10. Find the remainder when 2^{1000} is divided by 17.

(10 × 1 = 10)

Part B (Brief Answer Questions)*Answer any eight questions.**Each question carries 2 marks.*

11. If A and B are sets prove that $A - B = A \cap \bar{B}$.
12. Let $f, g: \mathbb{Z} \rightarrow \mathbb{Z}$ be such that $f(x) = 2x + 3$ and $g(x) = 3x + 2$. Find $(f \circ g)(1)$ and $(g \circ f)(1)$.
13. Give an example of a bijection from the set of all positive integers to the set of all odd positive integers. Justify your example.

Turn over

14. Define a directed graph. Draw the directed graph of the relation $\{(1,1), (1,2), (2,3), (3,1), (3,3)\}$.
15. Find the equivalence classes of the "congruence modulo 4" relation.
16. Prove that the divisibility relation is a partial order relation on the set of positive integers.
17. By constructing truth tables show that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.
18. Show that $\neg \forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent.
19. What is a direct proof? Use a direct proof to prove that the square of an even number is an even number.
20. If a is prime to b and each of these numbers is a divisor of N , prove that ab is a divisor of N .
21. Find the highest power of 7 contained in $\lfloor 2000 \rfloor$.
22. Prove that for any integer n , $n^5 - n$ is divisible by 10.

 $(8 \times 2 = 16)$

Part C (Short Essay Type Questions)

Answer any **six** questions.

Each question carries 4 marks.

23. State De Morgan's laws for sets. Show that $\overline{A \cup (B \cap C)} = (\bar{C} \cap \bar{B}) \cap \bar{A}$.
24. Let $f: B \rightarrow C$ and $g: A \rightarrow B$ be one to one and onto functions. Prove that $f \circ g$ is one to one and onto.
25. If x is a real number prove that $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$.
26. Explain how to construct the Hasse diagram of a partial order on a finite set. Illustrate with an example.
27. Prove that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.
28. Express the statement " $\lim_{x \rightarrow a} f(x) = L$ " using quantifiers.
29. Prove that the product of any n consecutive integers is divisible by $\lfloor n \rfloor$.
30. If m is prime to n , prove that $\varphi(mn) = \varphi(m)\varphi(n)$, where φ is the Euler's function.
31. If p is a prime of the form $4^m + 1$, prove that $\frac{1}{2}(p-1)$ is a solution of the congruence $x^2 + 1 \equiv 0 \pmod{p}$.

 $(6 \times 4 = 24)$

Part D (Essays)

Answer any **two** questions.

Each question carries 15 marks.

32. (a) Define the union, intersection, difference and symmetric difference of two sets A and B. Find these if $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7, 8\}$.
- (b) Is the function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2$ invertible? Justify your answer.
- (c) Prove that the set of positive rational numbers is countable.
33. (a) Prove that a relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, \dots$
- (b) List the ordered pairs in the equivalence relation R produced by the partition:
 $A_1 = \{1, 2, 3\}$, $A_2 = \{4\}$, $A_3 = \{5, 6\}$ of the set $S = \{1, 2, 3, 4, 5, 6\}$.
- (c) Obtain the maximal elements and minimal elements of the poset $\{2, 4, 5, 10, 12, 20, 25\}$ ordered by divisibility.
34. (a) Express the negation of the statement $\forall x \exists y(xy = 1)$ so that no negative precedes a quantifier.
- (b) Prove by contraposition that if $n = ab$ where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$.
- (c) Prove by contradiction that "if $3n + 2$ is odd, then n is odd".
35. (a) Prove that the sequence of primes is endless.
- (b) If $2^n + 1$ is a prime then prove that n is a power of 2.
- (c) State and prove Wilson's theorem.

(2 × 15 = 30)