

**B.Sc. DEGREE (C.B.C.S.) EXAMINATION, JUNE 2018**

**Second Semester**

**Complementary Course**

**MM2CMT03—Mathematics—DISCRETE MATHEMATICS—II**

(2017 Admissions only)

[Common to Computer Science M III and B.C.A.]

Time : Three Hours

Maximum : 80 Marks

**Part A**

*Answer any ten questions.  
Each question carries 2 marks.*

1. Find the number of edges in a graph with 10 vertices ?
2. Is  $K_4$  bipartite ? Why ?
3. Draw a graph with adjacency matrix :

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

4. Show that an undirected graph is a tree if and only if there is a unique simple path between the vertices.
5. Define a spanning tree of a graph. Also, show that a simple graph is connected if it has a spanning tree.
6. What is the ordered rooted tree that represents the expression  $((x + y) \uparrow 2 + (x - y)/3)$  ?
7. Find the values of the Boolean function represented by  $F(x, y, z) = xy + \bar{z}$ .
8. Find the sum of products expansion of the Boolean function  $f(x, y, z) = (x + z)y$ .
9. Write the dual of the Boolean expression  $x\bar{z} + x.0 + \bar{x}.1$ .

10. Express  $\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$  as sum of a symmetric matrix and skew symmetric matrix.

11. Find the characteristic equation of the matrix :

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & 0 & 1 \end{bmatrix}.$$

12. Find the rank of the matrix  $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix}$ .

(10 × 2 = 20 marks)

### Part B

*Answer any six questions.  
Each question carries 5 marks.*

13. Prove that an undirected graph has an even number of vertices of odd degree.
14. Prove that a simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same colour.
15. How many non-isomorphic simple graphs are there with three vertices ?  
Draw all of them.

16. Show that a full  $m$ -ary tree with  $l$  leaves has  $\frac{(ml - 1)}{(m - 1)}$  vertices and  $(l - 1)/(m - 1)$  internal vertices.

17. Using Huffman coding, encode the following symbols with the frequencies listed :

A	B	C	D	E	F
0.08	0.10	0.12	0.15	0.20	0.35

18. Show that  $x\bar{y} + y\bar{z} + \bar{x}z = \bar{x}y + \bar{y}z + x\bar{z}$ .

19. Show that  $\overline{(x \vee y)} = \bar{x} \wedge \bar{y}$  and  $\overline{(x \wedge y)} = \bar{x} \vee \bar{y}$ .

20. Reduce the matrix  $A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$  to normal form and also find the rank of A.

21. Solve  $x + y + z = 4$

$$2x - y + 2z = 5$$

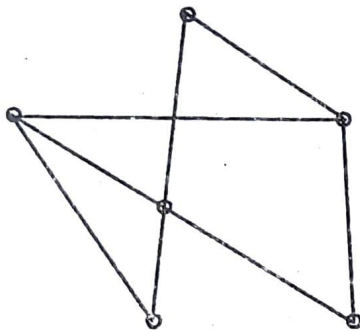
$x + y - z = 2$  using Cramer's rule.

(6 × 5 = 30 marks)

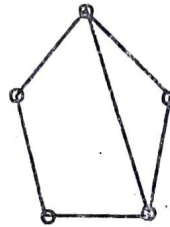
### Part C

*Answer any two questions.  
Each question carries 15 marks.*

22. (a) Determine whether the following graphs are isomorphic or not? Justify :



H



K

(b) Write an algorithm to find an Euler circuit in a graph having an Euler circuit.

23. (a) Write an algorithm to find a spanning tree from a connected graph.

(b) Consider the problem : "How is queens can be placed on an  $n \times n$  chessboard so that no two queens can attack one another". Explain how back tracking be used to solve the problem.

24. (a) Determine the characteristic roots and characteristic vectors of the matrix :

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

- (b) Verify Caylay Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

Hence or otherwise find  $A^{-1}$ .

25. (a) Design a circuit for a fixture controlled by three switches.

- (b) Construct a circuit that produce the output  $(x + y + z) \bar{x} \bar{y} \bar{z}$ .

(2 × 15 = 30 marks)