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QP CODE: 20000645

Reg No : ....

Name : ....

MSc DEGREE (CSS) EXAMINATION , NOVEMBER 2020

Second Semester

CORE - ME010204 - COMPLEX ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

BD25F00A

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Prove that an analytic function in a region Ω whose derivative vanishes identically must reduce to a constant.
2. Find the fixed points of the linear transformation $w = \frac{z}{2-z}$.
3. State Cauchy's theorem for a rectangle.
4. State Cauchy's theorem for a disk.
5. Define winding number of a closed curve γ with respect to a point.
6. State the Cauchy's integral formula for higher derivatives. Evaluate $\int_{|z|=2} z^{-4} sinz dz$.
7. Prove that the function $f(z)$ with a removable singularity at z = a can be extended to a unique analytic function at z = a.
8. Define the poles of a function. Give an example of a function having a triple pole.
9. State the general form of Cauchy's theorem.
10. Write a comment on Cauchy's principle value of an integral.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Show that z and z' correspond to diametrically opposite points on the Riemann sphere if and only if $z\overline{z'} = -1$.
12. Prove that a sequence of complex numbers is convergent if and only if it is a Cauchy sequence.
13. State and prove the necessary and sufficient conditions under which a line integral depends only on its end points.

14. Characterise rectifiable arcs.

15. Show that the function which is analytic in the whole plane and has a non essential singularity at $z = \infty$ reduces to a polynomial.

16. Let $f(z)$ be a nonconstant analytic function in a region Ω and has no zeros in Ω. Prove that $|f(z)|$ takes the minimum value on the boundary of Ω.

17. Prove that a region obtained from a simply connected region by removing n points has the connectivity n+1 and find a homology basis.

18. How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disc $|z| < 1$?

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i) Find the Linear Transformation which carries $0, i, -i$ into $1, -1, 0$.

(ii)Show that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

20. 1. State and prove the representation formula.
2. Compute $\int_{|z|=\rho} \frac{|dz|}{|z-a|^2}$, where $|a| \neq \rho$.

21. (a) State and prove the theorem on local correspondence.
(b) Prove that a nonconstant analytic function maps open sets onto open sets.

22. Let $f(z)$ be analytic except for isolated singularities a_j in a region Ω. Then prove that $\frac{1}{2\pi i} \int_\gamma f(z)dz = \Sigma_j n(\gamma, a_j) \times Res_{z=a_j} f(z)$, for any cycle γ which is homologues to zero in Ω and does not pass through any of the points a_j.

(2×5=10 weightage)